For Sem – IV   Paper - CC- IX

Class notes on Nuclear Physics
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Nuclear composition:

- Atomic nuclei are built up of protons and neutrons.
- Charge of a proton is $+1.6 \times 10^{-19}$ C and its mass is 1836 times greater than that of electron.
- Neutrons are uncharged particles and mass of a neutron is slightly greater than that of a proton.
- Neutrons and protons are jointly known as nucleons.
- Number of protons in nuclei of an element is equal to the number of electrons in neutral atom of that element.
- All nuclei of a given element does not have equal number of neutrons.
- Elements that have same number of protons but differ in number of neutrons in their nucleus are called **isotope.** For example, ordinary Hydrogen ($^{1}H_{1}$) has two isotopes i.e., Deuterium ($^{2}H_{1}$) and Tritium ($^{3}H_{1}$).
- Nuclei with same A but different Z ae known as **isobars**. For example, $^{40}K_{19}$ and $^{40}Ca_{20}$ share same mass number 40 but differs in one unit of Z.
- Nuclei with same number of neutrons but different number of protons are called **isotones**. For example, $^{198}Hg_{80}$ and $^{198}Au_{79.}$
- Symbol for nuclear species is written as $^{A}X_{Z}$ where X = Chemical symbol of element, Z = Atomic number of element or number of protons in the nucleus of that element and A = Mass number of nuclide or number of nucleons in the nucleus.

Atomic mass:

- Atomic masses refer to the masses of neutral atoms, not of bare nuclei i.e., an atomic mass always includes the masses of all its electrons.
- Atomic masses are expressed in mass units (u).
- One atomic mass unit is defined as $1/12^{th}$ part of the mass of $^{12}C_{6}$ atom.
- So the mass of $^{12}C_{6}$, the most abundant isotope of carbon is 12u.
- Value of a mass unit is $1u = 1.66054 \times 10^{-27}$ Kg
- Energy equivalent of mass unit are as follows
  From Einstein's Mass-energy relation $\Delta E = mc^2$
  Here, $\Delta m = 1.60 \times 10^{-27}$ Kg and $c = 3 \times 10^{8}$ m/s
  Therefore $\Delta E = (1.60 \times 10^{-27}) \times (3 \times 10^{8})^2$
  $= 1.49 \times 10^{-10}$ J
  But $1 \text{ eV} = 1.6 \times 10^{-19}$ J
  Therefore, $\Delta E = (1.49 \times 10^{-10}) / (1.60 \times 10^{-19})$
  $= 0.931 \times 10^{9}$ eV = 931 MeV
  Thus 1 amu = 931 MeV
- Mass of proton is 1.00727663u which is equal to 1.8725×10^{-27}Kg or 938.26 MeV.
- Mass of neutron is 1.0086654u which is equal to 1.6748×10^{-27}Kg or 939.55 MeV.

Size and shape of Nucleus:

- The size of an atom is about 10^{-10} m whereas the size of the nucleus of an atom is about 10^{-15}m which means it is about 10^{-5} (1/10000) of the size of the whole atom. The size of the nucleus is measured by the formula $R = R_0A^{1/3}$
  Where $R_0 = 1.4 \times 10^{-15}$ m
- Volume of the nucleus which is proportional to R^3 is proportional to A.
- Although the size of the nucleus is very small, the nucleus is massive compared to the rest of the atom. Typically, the nucleus contains more than 99.9% of the mass of the atom.
- All nuclei have the same density. The density of the nucleus is approximately 2.3×10^{17} kg/m^3.
  Calculation of density:
  Mass of each nucleon in a nucleus is 1.67×10^{-27}kg and $R_0 = 1.4 \times 10^{-15}$ m
  So, $M_N = A \times 1.67 \times 10^{-27}$ kg where A = mass number

  Nuclear Volume $V_N = 4/3 \times \pi \times [(R_0)^3 (A^{1/3})^3]$

  $= 4/3 \times \pi \times (R_0)^3A$

  Thus, nuclear density is $\rho_N = \frac{Mass\ of\ nucleus}{Nuclear\ volume}$

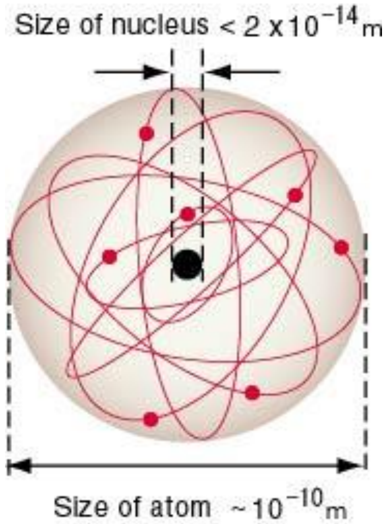
  $= \frac{A \, X \, 1.67 \, X \, 10^{-27}}{\frac{4}{3}\pi(R_0^3)A}$

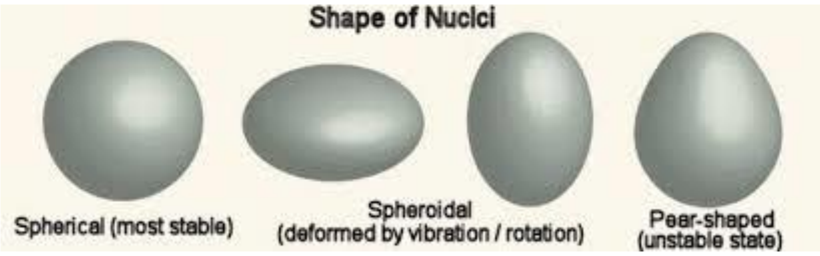
  $= \frac{3 \, X \, 1.67 \, X \, 10^{-27}}{12.56 \, X \, (1.4^3 \, X \, 10^{-45})}$

  $= \frac{5.01 \, X \, 10^{-27}}{34.46464 \, X \, 10^{45}}$

  $= 1.453 \times 10^{17}$ kg/m^3

- Nuclei are usually spherical in shape although some are spheroidal (egg-shaped).
- The proton and neutron are spherical, about 10^{-15}m in radius.





Non-existence of electrons in the nucleus as a consequence of Heisenberg Uncertainty Principle:

Let us assume that electrons exist n the nucleus. We know the radius of the nucleus is approximately 10^{-14} m and if electron exist inside the nucleus then uncertainty in the position of the electron is given by $\Delta x = 10^{-14}$ m

According to uncertainty principle $\Delta x . \Delta p_x = h/2\pi$

$$\text{Thus } \Delta p_x = h/2\pi \,.\, \Delta x$$

$$= \frac{6.62 \, X \, 10^{-34}}{2 \, X \, 3.14 \, X \, 10^{-14}}$$

$$= 1.05 \text{ x } 10^{-20} \text{ kg m/ sec}$$

If this is the uncertainty in the momentum of electron, then the momentum of electron should be at least of this order, i.e., $p = 1.05 \text{x} 10^{-20}$ kg m/ sec. An electron having this much high momentum must have a velocity comparable to the velocity of the light. Thus, its energy should be calculated by the following relativistic formula

$$E = \sqrt{({m_0}^2 c^4 + p^2 c^2)}$$

$$E = \sqrt{(9.1 \, X \, 10^{-31})^2 \, (3 \, X \, 10^8)^4 + \, (1.05 \, X \, 10^{-20})^2 \, (3 \, X \, 10^8)^2}$$

$$= \sqrt{9.9267 \, X \, 10^{-24}}$$

$$= 3.15 \text{x} 10^{-12} \text{ J}$$

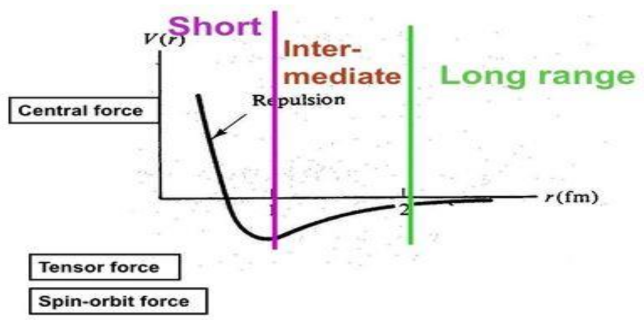
$$= \frac{3.15 \, X \, 10^{-12}}{1.6 \, X \, 10^{-19}} \, eV$$

$$= 19.6 \text{x} 10^6 \text{ eV or } 19.6 \text{ MeV}$$

Therefore, if the electron exists in the nucleus, it should have an energy of the order of 19.6 MeV. However, it is observed that β particles (electrons) ejected from the nucleus during β-decay have energies of approximately 3 MeV, which is quite different from the calculated value of 19.6MeV. Moreover, experimental results show that no electron or particle in the atom possess energy greater than 4 MeV. Therefore, it is confirmed that electrons do not exist inside the nucleus.

Nature of nuclear force:

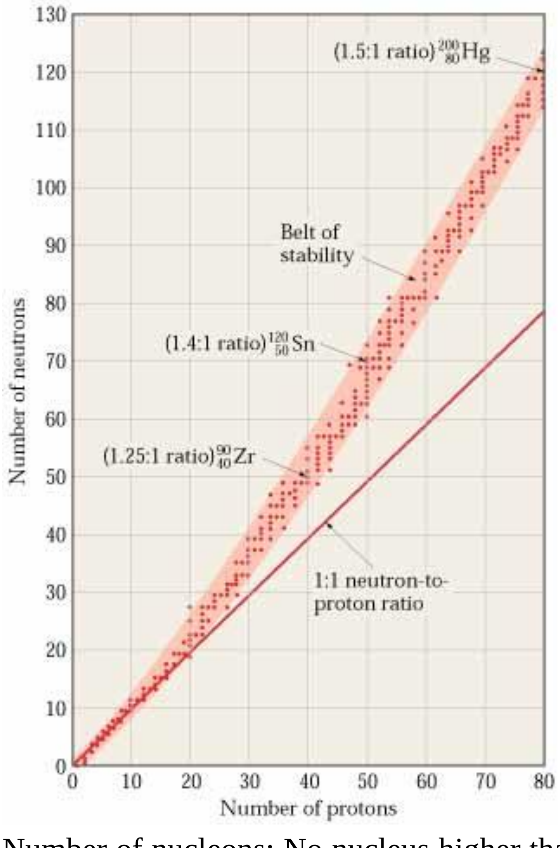
- **Nuclear Forces are strong attractive forces**: These are strongest forces and are about 10^{38} times stronger than gravitational forces. These are the forces which holds nucleons in a nucleus. It is attractive in nature but with a repulsive core. That is the reason that the nucleus is held together without collapsing in itself.
- **Nuclear forces are short range force:** The nuclear forces are most effective up to a distance of the order of 10^{-15} m or less. At 1 fermi, the distance between particles in a nucleus is extremely small. At this range, the nuclear force is much stronger than the repulsive Coulomb's force that pushes the particles away. These forces decrease rapidly as the separation distance between nucleons increases and becomes negligible when separation between nucleons is of the order of 2.5 fermi. The effective range up to which nuclear forces are effective is known as **nuclear range.**
- **Nuclear forces are not central forces:** These forces do not obey inverse square law. The magnitude of nuclear force depends on the direction of spin of nucleons.
- **Nuclear forces are saturated forces:** A nucleon experiences the force only due to its nearest neighbouring nucleons.
- **Nuclear force is identical for all nucleons:** It does not matter if it is a neutron or proton, once the Coulomb resistance is taken into consideration, nuclear force affects everything in the same way.
- **Nuclear force becomes repulsive at a distance of less than 0.7 fermi:** It is one of the most interesting properties of nuclear force that it is this repulsive component of the force that decides the size of the nulcleus. The nucleons come closer to each other till the point that the force allows, after which they can not come closer because of the repulsive property of the force.
- **Nuclear force has a very complex spin-dependence:** Evidence of this property first came from the observation that the deuteron deviates slightly from a spherical shape having a non-vanishing quadrupole moment. This suggests a force that depends on the orientation of the spins of the nucleons with respect to the vector joining the two nucleons which is as a **tensor force.** Moreover, in heavier nuclei, a shell structure has been observed which can be explained by a strong force between the spin of the nucleon and its orbital motion i.e., **spin orbit force**.



Nuclear stability and N-Z graph:

The two main factors that determine nuclear stability are the neutron-proton ratio and the total number of nucleons in the nucleus.

- Neutron-proton ratio: The principal factor for determining whether a nucleus is stable is the neutron to proton ratio. The graph below is a plot of the number of neutrons vs the number of protons in various stable isotopes. Stable nuclei with atomic numbers up to about 20 have an n:p ratio of about 1:1. Above Z=20, the number of neutrons always exceeds the number of protons in sable isotopes. The stable nuclei are located in the pink band as the zone of stability. As the atomic number increases, the zone of stability corresponds to a gradually increasing neutron/proton ratio. In the case of the heaviest stable isotope, $^{209}Bi_{83}$ for instance, the neutron/proton ratio is 1.518. If an unstable isotope lies to the left of the zone of stability it is neutron rich and decays by β emission. If it lies to the right of the zone, it is proton rich and decays by positron emission or electron capture.



- Number of nucleons: No nucleus higher than Pb-208 is stable. This is because although the nuclear strong force is about 100 times strong as the electrostatic repulsions, it operates over only very short distances. When a nucleus reaches a certain size, the strong force is no longer able to hold the nucleus together. Another factor affecting the stability of a nucleus is whether the number of protons and neutrons is even or odd. Among the 354 known stable isotopes, 157 (almost half) have an even number of protons and an even number of neutrons. Only five have an odd number of both kinds of nucleons. In the universe as a whole (with the exception of hydrogen) we find that the even-numbered elements are almost always much more abundant than the odd-numbered elements.

- Finally, there is a particular stability associated with nuclei in which either the number of protons or the number of neutrons is equal to one of the so-called "magic" numbers 2, 8, 20, 28, 50, 82, and 126. These numbers correspond to the filling of shells in the structure of the nucleus. These shells are similar in principle but different in detail to those found in electronic structure. Examples are $^{4}He_{2}$, $^{16}O_{8}$, $^{40}Ca_{20}$ and $^{208}Pb_{82}$.

Liquid drop model: Semi-empirical mass formula and binding energy:

According to this model, the atomic nucleus behaves **like the molecules in a drop** of liquid. But in this nuclear scale, the fluid is made of nucleons (protons and neutrons), which are held together by **the strong nuclear force**. The liquid drop model of the nucleus takes into account the fact that the nuclear forces on the nucleons on the surface are different from those on nucleons in the interior of the nucleus. The **interior nucleons are completely surrounded** by other attracting nucleons. Here is the analogy with the forces that form a drop of liquid.

The semi-empirical mass formula (also called the Bethe-Wiezsacker's mass formula) is an expression for binding energy of a nucleus assuming the liquid drop model.
The full formula takes the form,

$$E_B = a_1A - a_2A^{2/3} - a_3Z(Z-1)/A^{1/3} - a_4(A-2Z)^2/A + \delta(A,Z)$$

1) The 1st term is called the **volume energy term**:
Since, volume energy, $E_V \propto V$ and $V \propto R^3$
But, $R = R_0A^{1/3}$
This gives, $E_V \propto A \Rightarrow E_V = a_1A$

2) The 2nd term is the **surface energy term:**
The surface term is a correction to the volume term due to the fact that nucleons on the surface of the nucleus interact only with nucleons on one side, unlike the interior nucleons which are attracted equally from all sides.
Now, $E_S \propto S$ and $S \propto R^2$
But, $R \propto A^{1/3}$ and this gives,
$E_S = -a_2A^{2/3}$ where the negative sign indicates that the surface energy contributes to decreasing the binding energy.

3) The 3rd term is the **Coulomb energy term.**
The Coulombic interaction between protons destabilizes the nucleus and hence contributes to a decrease in the binding energy.
Since, each proton is repelled by $(Z-1)$ other protons, hence there are $Z(Z-1)/2$ repelling pairs. (the factor of 1/2 is to avoid double counting of pairs).
Also, since Coulombic potential energy is inverse proportional to $R \propto A^{1/3}$ and hence,
$E_C \propto Z(Z-1)/A^{1/3} = -a_3Z(Z-1)/A^{1/3}$

4) The 4th term is the **asymmetry term**.

The maximum stability occurs for Z = N. Any departure from this introduces an asymmetry energy which again tends to destabilize and hence reduce the binding energy (hence a negative sign).
The symmetry energy is directly proportional to the neutron excess (N−Z) and also the fraction of nuclear volume in which it is contained.
Thus, $E_A \propto (N-Z)$ and $E_A \propto (N-Z)/A$
Therefore, the symmetry term takes the form,
$E_A = -a_4(A-2Z)^2/A$ where, N−Z = A−Z−Z = A−2Z

5) The 5th term is the **pairing energy** among nucleons and can take both negative and positive values depending on the particular case.
Hence, the binding energy expression takes the form

$$E_b(MeV) = a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z^2}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} \pm \delta(A,Z)$$

$$\delta(A,Z) = \begin{matrix} +\delta_0 \; for \; Z, N \; even \\ 0 \\ -\delta_0 \; for \; Z, N \; odd \end{matrix}$$

With the aid of **the Weizsaecker formula** the binding energy can be calculated very well for nearly all isotopes. This formula provides a good fit for heavier nuclei. For light nuclei, especially for $^4He_2$, it provides a poor fit. The main reason is the formula does not consider the internal shell structure of the nucleus.
In order to calculate the binding energy, the coefficients a_V, a_S, a_C, a_A and a_P must be known. The coefficients have units of **megaelectronvolts (MeV)** and are calculated **by fitting** to **experimentally measured masses of nuclei**. They usually vary depending on the fitting methodology. According to ROHLF, J. W., the coefficients in the equation are following:

$$E_b(MeV) = 15.76A - 17.81A^{2/3} - 0.711\frac{Z^2}{A^{1/3}} - 23.7\frac{(N-Z)^2}{A} \pm 34A^{-3/4}$$

Mass Defect:

A mass defect is the difference between an atom's mass and the sum of the masses of its protons, neutrons, and electrons. The reason the actual mass is different from the masses of the components is because some of the mass is released as energy when protons and neutrons bind in the atomic nucleus. Thus, the mass defect results in a lower-than-expected mass. The mass defect follows the conservation laws, where the sum of mass and energy of a system is constant, but matter can be converted into energy.
The mass defect can be calculated using the following equation. In calculating the mass defect, it is important to use the full accuracy of mass measurements because the difference in mass is small compared to the mass of the atom. Rounding off the masses of atoms and particles to three or four significant digits prior to the calculation will result in a calculated mass defect of zero.
$\Delta m = [Z(m_p + m_e) + (A-Z)m_n] - m_{atom}$

where

Δm = mass defect (amu)

m_p= mass of a proton (1.007277 amu)

m_n= mass of a neutron (1.008665 amu)

m_e= mass of an electron (0.000548597 amu)

m_{atom}= mass of nuclide ^XZ_A (amu)

Z = atomic number (number of protons)

A = mass number (number of nucleons)

Example:

Calculate the mass defect for lithium-7. The mass of lithium-7 is 7.016003 amu.

Solution:

$\Delta m = [Z(m_p + m_e) + (A-Z)m_n] - m_{atom}$

$= [3(1.007277 + 0.000548597) + (7-3)1.008665] - 7.016003$ amu

$= 0.0421335$ amu

Nuclear Binding energy:

When the protons and neutrons combine to form a nucleus, the mass that disappears (mass defect, Δm) is converted into an equivalent amount of energy (Δmc^2). This energy is called the binding energy of the nucleus.

$$\text{Binding energy} = [Zm_P + Nm_n - m]\, c^2$$

$$= \Delta m\, c^2$$

Calculation of Binding Energy:

Since the mass defect was converted to BE (binding energy) when the nucleus was formed, it is possible to calculate the BE using a conversion factor derived by the mass–energy relationship from Einstein's Theory of Relativity. Einstein's famous equation relating mass and energy is $E = mc^2$ where *c* is the velocity of light (*c*=2.998×108 m/sec). The energy equivalent of 1 amu can be determined by inserting this quantity of mass into Einstein's equation and applying conversion factors.

$E = mc^2$

$= 1$ amu

$= 1 \text{ amu } (1.6606 \text{ X } 10^{-27} \text{ kg/1 amu}) (2.998 \text{ X } 10^{8} \text{ m/sec})^2 \text{ X } (1\text{N/kg m /sec}^2) (1\text{J/1N m})$

$= 1.4924 \text{ X } 10^{-10} \text{ J } (1 \text{ MeV}/1.6022 \text{ X } 10^{-13} \text{ J}) (2.998 \text{ X } 10^{8} \text{ m/sec})$

$= 931$MeV

Conversion Factors used

1 amu = 1.6606×10^{-27} kg
1 N = 1 kg m/sec^2
1 J = 1 Nm
1 MeV = 1.6022×10^{-13} J

Since 1 amu is equivalent to 931.5 MeV of energy, the BE can be calculated as

BE = Δm (931.5 MeV/1amu)

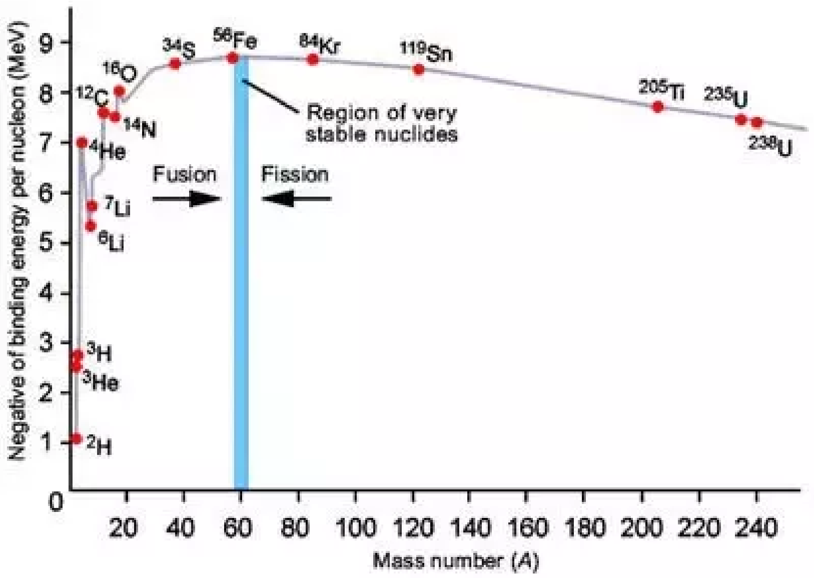
In the above example of lithium-7, the mass defect Δm = 0.0421335 amu so its binding energy is BE = Δm (931.5 MeV/1amu)

= 0.0421335 amu (931.5 MeV/ 1 amu)

= 39.24 MeV

The binding energy of a nucleus determines its stability against disintegration. In other words, if the binding energy is large, the nucleus is stable and vice versa. The binding energy per nucleon is BE/ A = Binding energy of the nucleus / Total number of nucleons

It is found that the binding energy per nucleon varies from element to element. A graph is plotted with the mass number A of the nucleus along the X−axis and binding energy per nucleon along the Y-axis (Fig).



Explanation of binding energy curve:

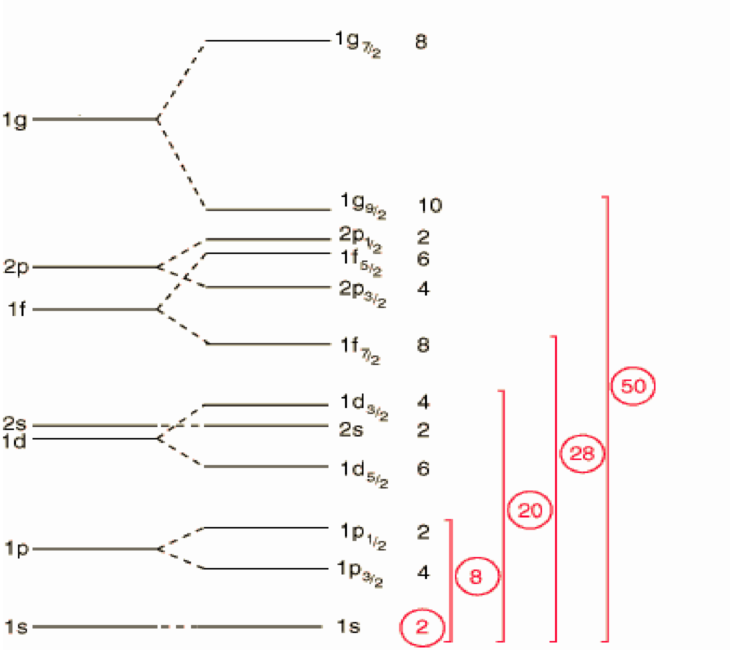
- The binding energy per nucleon increases sharply with mass number A upto 20. It increases slowly after A = 20. For A<20, there exists recurrence of peaks corresponding to those nuclei, whose mass numbers are multiples of four and they contain not only equal but also even number of protons and neutrons. Example: $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$, and $_{10}Ne^{20}$. The curve becomes almost flat for mass number between 40 and 120. Beyond 120, it decreases slowly as A increases.
- The binding energy per nucleon reaches a maximum of MeV at A=56, corresponding to the iron nucleus ($_{26}Fe^{56}$). Hence, iron nucleus is the most stable.
- The average binding energy per nucleon is about 8.5 MeV for nuclei having mass number ranging between 40 and 120. These elements are comparatively more stable and non-radioactive.
- For higher mass numbers the curve drops slowly and the BE/A is about 7.6 MeV for uranium. Hence, they are unstable and radioactive.
- The lesser amount of binding energy for lighter and heavier nuclei explains nuclear fusion and fission respectively. A large amount of energy will be liberated if lighter nuclei are fused to form heavier one (fusion) or if heavier nuclei are split into lighter ones (fission).

Nuclear Shell Model and magic numbers:

In nuclear physics, the **nuclear shell model** is a theoretical model proposed by Dmitry Ivanenko in 1932 to describe the atomic nucleus. It must be noted this model is based on the Pauli exclusion principle to describe the structure of the nucleus in terms of energy levels. The **nuclear shell model** is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. **Nucleons** are added to shells which increase with energy that orbit around a central potential. In the atomic shell model the central potential around which the electrons orbit is generated by the nucleus. Nucleons are added to shells which increase with energy that orbit around a central potential.

In comparison to atomic shell model, the **atomic nucleus** governed by two different forces. The residual strong force, also known as the nuclear force, acts to hold neutrons and protons together in nuclei. In nuclei, this force acts against the enormous repulsive electromagnetic force of the protons. The term residual is associated with the fact that it is the residual of the fundamental strong interaction between the quarks that make up the protons and neutrons. The strong interaction is very complicated interaction, because it significantly varies with distance. At distances comparable to the diameter of a proton, the strong force is approximately 100 times as strong as electromagnetic force. **At smaller distances**, however, the strong force between quarks becomes **weaker**, and the quarks begin to behave like independent particles. In particle physics, this effect is known as **asymptotic freedom**.
With the enormous strong force acting between individual nucleons and with so many nucleons to collide with, how can nucleons orbit a central potential without interacting? This problem is explained by the **Pauli exclusion principle**, which states that two fermions cannot occupy the same quantum state. In other words, the interaction will not occur, if the higher energy shells are fully occupied and the energy imparted to the nucleon during the collision is insufficient to promote the nucleon to an unfilled orbit. As a result, the nucleons orbit becomes independent of one another. The nuclear shell model was able to describe many phenomena like the **magic numbers**, the ground state spin and parity etc.

Relation between shell model and magic numbers



Magic Numbers of Protons and Neutrons:

A **magic number** is a number of nucleons in a nucleus, which corresponds to complete shells within the atomic nucleus. Atomic nuclei consisting of such a magic number of nucleons have a higher average binding energy per nucleon than one would expect based upon predictions such as the mass formula of **von Weizsaecker** (also called the semi-empirical mass formula - SEMF) and are hence more stable against nuclear decay. **Magic numbers** are predicted by the nuclear shell model and are proved by observations that have shown that there are sudden discontinuities in the proton and neutron separation energies at specific values of Z and N. These correspond to the closing of shells (or sub-shells). Nuclei with closed shells are more tightly bound than the next higher number. The closing of shells occurs at Z or N = 2, 8, 20, 28, 50, 82, 126. It is found that nuclei with even numbers of protons and neutrons are more stable than those with odd numbers. Nuclei which have both neutron number and proton number equal to one of the **magic numbers** can be called "**doubly magic**", and are found to be particularly stable.

$$Doubly\ Magic \quad {}^{4}_{2}He \quad {}^{16}_{8}O \quad {}^{40}_{20}Ca \quad {}^{48}_{20}Ca \quad {}^{208}_{82}Pb$$

There are further special properties of nuclei, which have a magic number of nucleons:

- Higher abundance in nature. For example, helium-4 is among the most abundant (and stable) nuclei in the universe.
- The stable elements at the end of the decay series all have a "magic number" of neutrons or protons. The nuclei He-4, O-16, and Pb-208 (82 protons and 126 neutrons) that contain magic numbers of both neutrons and protons are particularly stable. The relative stability of these nuclei is reminiscent of that of inert gas atoms (closed electron shells).
- Nuclei with N = magic number have much lower neutron absorption cross-sections than surrounding isotopes.
- These nuclei appear to be perfectly spherical in shape; they have zero quadrupole electric moments.
- Magic number nuclei have higher first excitation energy.